

WE START OUT with a recapitulation of the basic notions of dynamics. Our aim is narrow; we keep the exposition focused on prerequisites to the applications to be developed in this text. We assume that the reader is familiar with dynamics on the level of the introductory texts mentioned in remark 1.1, and concentrate here on developing intuition about what a dynamical system can do. It will be a coarse brush sketch—a full description of all possible behaviors of dynamical systems is beyond human ken. While for a novice there is no shortcut through this lengthy detour, a sophisticated traveler might bravely skip this well-trodden territory and embark upon the journey at chapter 18.

The fate has handed you a flow. What are you to do about it?

1. Define your *dynamical system*  $(\mathcal{M}, f)$ : the space of its possible states  $\mathcal{M}$ , and the law  $f^t$  of their evolution in time.
2. Pin it down locally—is there anything about it that is stationary? Try to determine its *equilibria / fixed points* (chapter 2).
3. Cut across it, represent as a map from a section to a section (chapter 3).
4. Explore the neighborhood by *linearizing* the flow—check the *linear stability* of its equilibria / fixed points, their stability eigen-directions (chapters 4 and 5).
5. Does your system have a *symmetry*? If so, you must use it (chapters 10 to 12). Slice & dice it (chapter 13).
6. Go global: train by *partitioning the state space* of 1-dimensional maps. Label the regions by *symbolic dynamics* (chapter 14).
7. Now venture global distances across the system by continuing eigenvectors into *stable / unstable manifolds*. Their intersections *partition the state space* in a dynamically invariant way (chapter 15).
8. Guided by this topological partition, compute a set of *periodic orbits* up to a given topological length (chapter 16).

Along the way you might want to learn about dynamical invariants (chapter 5), Lyapunov exponents (chapter 6), classical mechanics (chapter 8), billiards (chapter 9), finite groups (chapter 10), and discrete (chapter 11) and continuous (chapter 12) symmetries of dynamics.